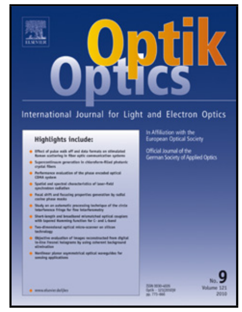

Title: A novel weak signal detection method for Linear Frequency Modulation signal based on Bistable system and Fractional Fourier Transform

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A novel weak signal detection method for Linear Frequency Modulation signal based on Bistable system and Fractional Fourier Transform

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Abstract

Fractional Fourier Transform (FRFT) is regarded as an effective method for detection of Linear Frequency Modulation (LFM) signals in recent years. However, the performance of FRFT detection will deteriorate sharply in the weak noise scene. An alternative solution is the use of bistable system, which can generate stochastic resonance (SR), and is relatively easy to achieve. This paper proposes a novel LFM weak signal detection algorithm based on bistable system and FRFT, named BSFRFT. We use the SR effect of bistable system to amplify LFM signal, and then apply FRFT to the previous output. We also present an evaluation criteria to measure the ability of detection methods and to compare performance of FRFT and BSFRFT. Numerical simulations show that the algorithm is effective for two cases – the LFM signal covered by white noise or colored noise (including pink noise).

Keywords: Bistable system, Linear Frequency Modulation signal, Fractional Fourier Transform, Weak signal detection, BSFRFT

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1 Introduction

Linear Frequency Modulation signal (LFM signal), which is always implemented as linear chirp signal, is widely used in sonar and radar systems. It could get larger compression ratio as well as having excellent range resolution and radial velocity resolution ration [1].

The typical LFM signal can be written as

$$S(t) = A_0 \cos(2\pi\mu t^2 + 2\pi f_0 t), \tag{1}$$

where A_0 is the signal amplitude, μ the rate of frequency increase or chirp rate, and f_0 the starting frequency.

There are many methods to estimate LFM signal due to their time-varing characteristic, such as short-time Fourier transform (STFT) [2], Wigner-Ville distribution (WVD) [3], Radon-Wigner transform (RWT) [4], Wigner-Hough transform (WHT) [5] and so on. However, the above approaches have some disadvantages. The STFT cannot achieve a satisfied time-frequency resolution due to the window function. The WVD suffers from the effect of cross-terms. Although some techniques have been proposed to suppress the cross-terms, its time-frequency resolution is then reduced and the computational complexity increased [6]. The RWT and WHT are both two-dimensional search algorithms, whose searching time is large and cross-terms interferences also exist [1].

Fractional Fourier Transform (FRFT) is a generalization of the conventional Fourier Transform. The LFM signal has the best characteristics of energy concentration in certain specific fractional Fourier domain [1], then FRFT provides a higher time-frequency resolution than STFT and it avoids the effect of cross-terms produced by the WVD [6]. Due to its orthonormal chirped basis, the LFM signal fits well in the FRFT domain for detection and estimation [6]. However, through the theoretical analysis of FRFT detection performance, it is noted that although FRFT is a linear transformation, the output SNR relative to the input SNR has a threshold effect – when the input SNR is higher than the threshold, the output SNR is improved and when the input SNR is below the threshold, the output SNR is worsened [10]. In other words, when input SNR is not large enough, the FRFT detection result is not satisfactory.

Stochastic resonance (SR) is a phenomenon wherein the response of a nonlinear system to a weak periodic or aperiodic input signal is optimized by the presence of a particular level of noise [8], and broadly adopted to describe any phenomenon where the presence of noise in a nonlinear system is better for output signal quality than its absence [7]. Bistable system is a simple model used to generate SR and has received much research attention. It has two stable equilibrium states and can transition from one state to the other if it is given enough activation energy to penetrate the barrier. Then SR effect of bistable system has simple physical explanation, that is, with the help of random forces, a particle makes ocasional transitions from an equilibrium state over the barrier in the center and as the input noise variance is increased, the rate at which such jumps will occur increases, but once noise variance is large enough that the barrier becomes easy to surmount, the rate grows more slowly as the noise is further increased [9].

In this paper, we will use bistable system to amplify the input LFM signal covered by noise with the SR effect and then apply FRFT to detecting the output of bistable system.

2 LFM signal detection based on FRFT

2.1 Basis of FRFT

The FRFT of signal $x(t)$ is represented as

$$X_p(u) = F^p[x(t)] = \int_{-\infty}^{\infty} x(t) K_\alpha(t,u) dt, \tag{2}$$

where p is the fractional order, $\alpha = \frac{p\pi}{2}$, $F^p[\cdot]$ denotes the FRFT operator, and $K_\alpha(t,u)$ is the kernel of the FRFT:

$$K_\alpha(t,u) = A_\alpha exp(j\pi(u^2 \cot\alpha - 2ut\csc\alpha + t^2\cot\alpha)) \tag{3}$$

with $A_\alpha = \sqrt{1 - j\cot\alpha}$. $K_\alpha(t,u)$ has the following properties:

$$K_{-\alpha}(t,u) = K_\alpha^*(t,u), \tag{4}$$

$$\int_{-\infty}^{\infty} K_\alpha(t,u) K_\alpha^*(t,u^{'}) = \delta(u - u^{'}). \tag{5}$$

Hence, the inverse FRFT is

$$x(t) = F^{-p}[X_p(u)] = \int_{-\infty}^{\infty} X_p(u) K_{-\alpha}(t,u) du. \tag{6}$$

Equation (6) indicates that signal $x(t)$ can be interpreted as a decomposition to a basis formed by the orthonormal LFM functions in the u domain, and the u domain is usually called the fractional Fourier domain, in which the time and frequency domains are its special cases [11].

2.2 Discrete FRFT Algorithm

Ozaktas gave an effective DFRFT algorithm [12]:

$$X_p(u) = \frac{A_\alpha}{2F} \sum_{n=-N}^{N} exp(j\pi u^2 \cot\alpha) exp(\frac{j\pi n^2 \cot\alpha}{(2F)^2} - \frac{j2\pi un \csc\alpha}{2F}) s(\frac{n}{2F}), \tag{7}$$

where p, α, A_α are defined as in Subsection 2.1, F is the highest frequency of signal $s(t)$, n is the sampling point for t, $N = F^2$, and $s(t)$ is assumed to be zero outside $[-\frac{F}{2}, \frac{F}{2}]$.

Go on to discrete u, we get that:

$$X_p(\frac{m}{2F}) = \frac{A_\alpha}{2F} \sum_{n=-N}^{N} exp(\frac{j\pi m^2 \cot\alpha}{(2F)^2} + \frac{j\pi n^2 \cot\alpha}{(2F)^2} - \frac{j2\pi mn \csc\alpha}{(2F)^2}) s(\frac{n}{2F}), \tag{8}$$

where m is the sampling point for u.

After some algebraic manipulations, we can rewrite (8) as the following form:

$$\begin{aligned} & X_p(\frac{m}{2F}) \\ = & \frac{A_\alpha}{2F} exp(j\pi(\cot\alpha - \csc\alpha)(\frac{m}{2F})^2) \\ & \sum_{n=-N}^{N} exp(j\pi \csc\alpha(\frac{m-n}{2F})^2) exp(j\pi(\cot\alpha - \csc\alpha)(\frac{n}{2F})^2) s(\frac{n}{2F}). \end{aligned} \tag{9}$$

It can be recognized that the summation is the convolution of $exp(j\pi \csc\alpha(\frac{n}{2F})^2)$ and the chirp-modulated function $s(\cdot)$. The convolution can be computed in $O(NlogN)$ time by using the FFT. Hence, the overall complexity is $O(NlogN)$ [12].

We will apply the above numerical algorithm to computing the FRFT of LFM signal and LFM signal plus Gaussian white noise, where fractional order p is from 0 to 2. In order to get various SNR of LFM signal and noise, we only change the variance D of Gaussian white noise $\xi(t)$ and fix parameters of LFM signal $S(t)$. For equation (1), let A_0 be $0.3162(-10dB)$, μ be 0.0001 and f_0 be 0.01 respectively. The figures of $S(t)$, $\xi(t)$ and $S(t) + \xi(t)$ (SNR$=-20dB$) are given in Figure 1-3.

It's obvious that LFM signal is covered totally by noise if you compare Figure 2 and Figure 3.

The FRFT output of LFM signal described in equation (1) of various fractional orders (step value is 0.005) is shown in Figure 4.

We can find the best fractional order which maximizes the module of FRFT output is $p = 0.9950$ (step value is 0.001), the module figure of u domain corresponding to the best order is shown in Figure 5. In order

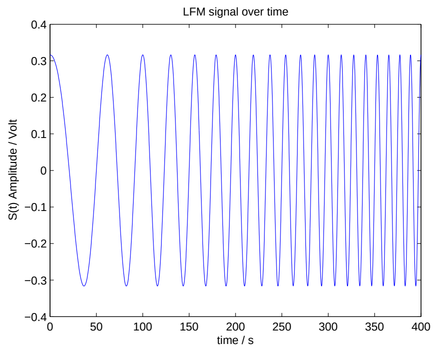

Figure 1: LFM signal

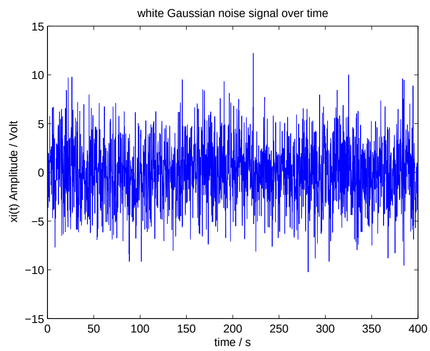

Figure 2: Gaussian white noise

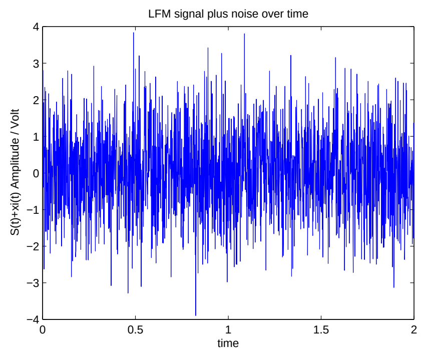

Figure 3: LFM signal plus Gaussian white noise

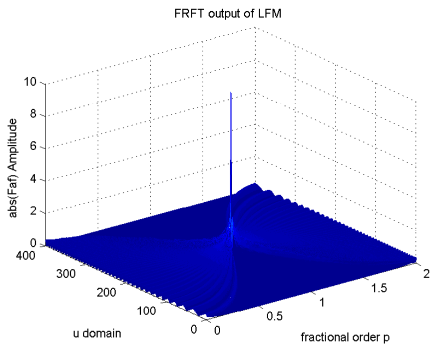

Figure 4: FRFT output of LFM signal

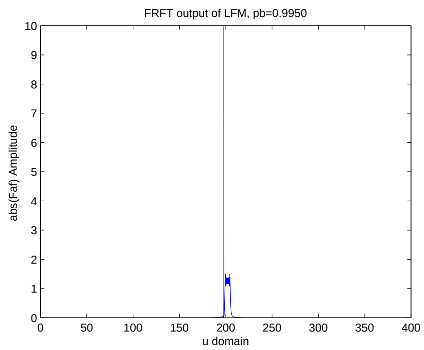

Figure 5: FRFT output of LFM signal for best fractional order

to illustrate the invalidity of FRFT detection method when SNR is weak ($-20dB$), we can refer to Figure 6. It's obvious that the FRFT output has no apparent peak point to differentiate signal from noise.

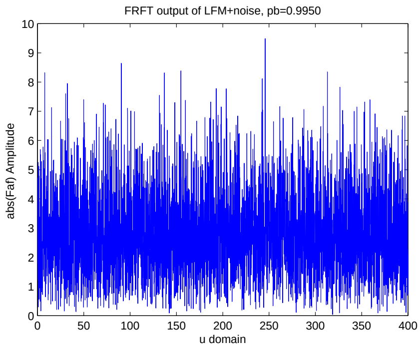

Figure 6: FRFT output of LFM signal plus heavy white noise

3 SR driven by LFM input signal through bistable system

The classical model of bistable (BS) system is written as

$$\dot{x}(t) = -\dot{U}(x) + S(t) + \xi(t), \tag{10}$$

where $\dot{U}(x) = -0.5ax^2 + 0.25bx^4$ is the potential function, $a, b > 0$, $\xi(t)$ is Gaussian white noise with zero mean and autocorrelation $< \xi(t)\xi(s) >= 2D\delta(t-s)$, the brackets $< \cdot >$ denote an ensemble average, and $S(t)$ is a signal, specially, it is LFM signal as described in equation (1) in this paper. Let $S(t) + \xi(t)$ be 0, then it's easy to show that the two stable equilibrium points of equation (10) are $\sqrt{\frac{a}{b}}$ and $-\sqrt{\frac{a}{b}}$.

We can use Runge-Kutta algorithm to compute $x(t)$ in (10),

$$C(n+1) = S(n+1) + \xi(n+1), \tag{11}$$

$$k_{x1} = h[-bx(n)^3 + ax(n) + C(n+1)], \tag{12}$$

$$k_{x2} = h[-b(x(n) + \frac{k_{x1}}{2})^3 + a(x(n) + \frac{k_{x1}}{2}) + C(n+1)], \tag{13}$$

$$k_{x3} = h[-b(x(n) + \frac{k_{x2}}{2})^3 + a(x(n) + \frac{k_{x2}}{2}) + C(n+1)], \tag{14}$$

$$k_{x4} = h[-b(x(n) + k_{x3})^3 + a(x(n) + k_{x3}) + C(n+1)], \tag{15}$$

$$x(n+1) = x(n) + \frac{1}{6}(k_{x1} + 2k_{x2} + 2k_{x3} + k_{x4}), \tag{16}$$

where h is step value, $S(n)$,$\xi(n)$,$x(n)$ are sampling values of $S(t)$,$\xi(t)$,$x(t)$ respectively.

Let $a = 1, b = 1, h = 0.1$. When input SNR is $-15dB$, the output signal $x(t)$ from bistable system is shown as in Figure 7. Its tendency is like the input signal and has a larger amplitude, which indicates stochastic resonance really takes place. When noise is weak, we take $0dB$ SNR as an example. If

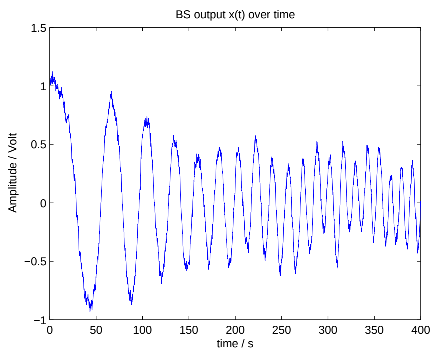

Figure 7: Output from bistable system(noise=5dB)

the initial value of $x(n)$ is 1, the output signal centers on equilibrium point 1 and doesn't have amplitude amplification effect (as shown in Figure 8). Similarly, if the initial value of $x(n)$ is -1, the centered equilibrium point is -1 correspondingly.

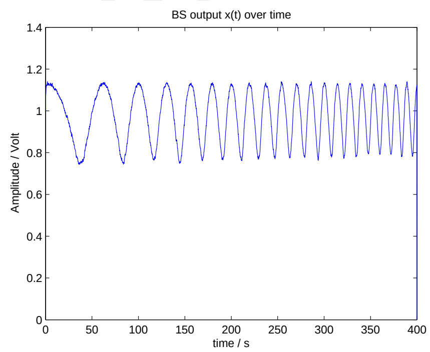

Figure 8: Output from bistable system(noise=-10dB)

4 Novel method used to detect weak LFM signal

From above sections, we note that the bistable system has ability to amplify the original LFM signal in noise. At the same time, FRFT is useful in handling with LFM signal detection in heavy SNR. Therefore, we propose a novel method to detect weak LFM signal covered by heavy noise, as shown in Figure 9. For convenience, we name this method BSFRFT.

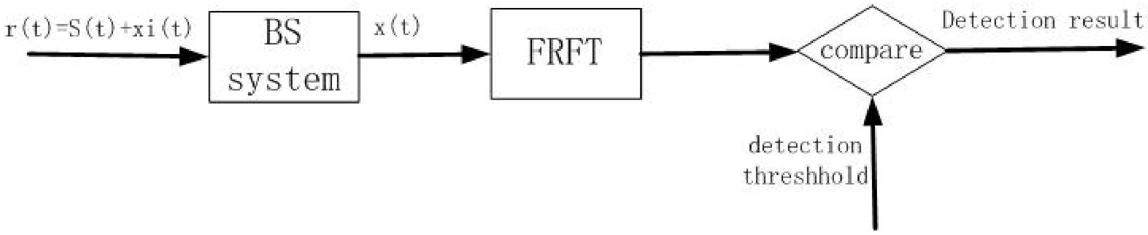

Figure 9: Schematic illustration of BSFRFT

First, we use the stochastic resonance effect of bistable system to amplify LFM signal, and then take FRFT transform to the previous output. However, very weak noise which is away from the best SR point has little role in amplifying signal, thus BS system has no positive act in this situation. Meanwhile, when noise is weak and input of BS system is only noise, we note that the output signal centers on equilibrium point 1 and has no enough energy to transition to another equilibrium point -1. In other words, the output from BS system may have direct-current (DC) component. The FRFT output of a DC signal also has a larger peak point, then it is not convenient for us to differentiate LFM signal from noise by FRFT in this situation. So we give the modified form detection method, as shown in Figure 10. We name it MBSFRFT.

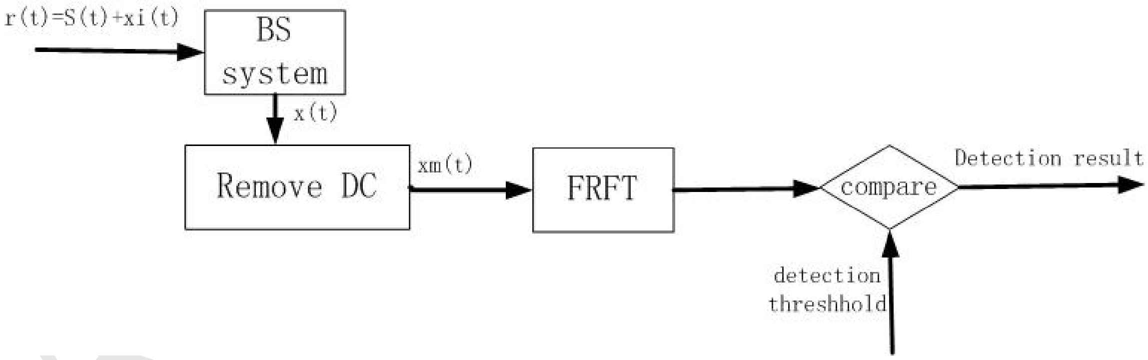

Figure 10: Schematic illustration of MBSFRFT

We should remove the DC component of BS system output before doing FRFT to differentiate signal from noise.

(1) How to check the existence of DC component?

Let $x(t)$ be the output from BS system. Because the two stable equilibrium points of equation (10) are 1 and -1, the DC component of $x(t)$ must be 1 or -1. Let $MT < 1$ be a positive number, we have the DC check formula:

$$EDC1 = (mean(x) - 1) \leq MT \&\& (mean(x) - 1) \geq -MT, \quad (17)$$

$$EDC2 = (mean(x) + 1) \leq MT \&\& (mean(x) + 1) \geq -MT. \quad (18)$$

If EDC1 holds, we assert that $x(t)$ has DC component 1, if EDC2 holds, we assert that $x(t)$ has DC component -1.

(2) Remove DC component.

Let $xm(t)$ be signal to FRFT. Then if EDC1 holds, $xm(t) = x(t) - 1$, if EDC2 holds, $xm(t) = x(t) + 1$, otherwise $xm(t) = x(t)$.

A direct question is – how weak signal can be detected by BSFRFT or MBSFRFT? A bigger question is – what's the signal extent of signals which can be detected? We will present an evaluation criteria for FRFT detection here, actually, it can be applied in a wider field for other detection methods. Let $X_{pb}(u)$ be the FRFT output of LFM signal plus noise and $|\xi_{pb}(u)|$ be the FRFT output of noise. Because FRFT of LFM signal is an impulse function and FRFT of white noise is distributed uniformly in u domain, the maximum of $|X_{pb}(u)|$ should be output of LFM component and other points be output of noise as long as the noise output is not larger than LFM output. When noise power increases larger and larger, the max FRFT output of noise maybe surpass output of signal, so the gap between maximum of $|X_{pb}(u)|$ and $|\xi_{pb}(u)|$ can indicate ability of FRFT. We define a variable G to measure effect of FRFT

$$G = \max_u |X_{pb}(u)| - \max_u |\xi_{pb}(u)|, \quad (19)$$

where pb is the best fractional order for LFM signal. We can give the noise-varying image (step is $0.5dB$) under 100 times Monte Carlo simulation, as shown in Figure 11.

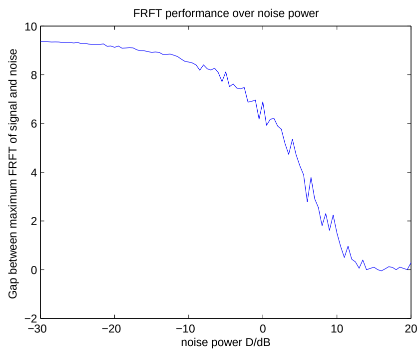

Figure 11: FRFT G versus noise power D

The $G - D$ figure is monotonically decreasing, that is consistent with our impression – input SNR is larger and detection result of FRFT is better.

When noise power is larger than $10dB$ (or SNR is less than $-20dB$), FRFT detection method does not work anymore. We have got the FRFT output of SNR=$-20dB$ before in Figure 6 which fits the analysis well.

It is a different case when signal is filtered by bistable system before doing FRFT, we can discuss this situation and answer the above question in the next section.

5 Simulation of the method

5.1 Simulation of additive Gaussian white noise

In this section, we will discuss the ability of BSFRFT/MBSFRFT and show some numerical results when the background noise is Gaussian white noise. The FRFT of $x(t)$, which is got from Section 3 (SNR=$-15dB$), is shown in Figure 12. Correspondingly, Figure 13 gives the result when the input of BS system is white noise only.

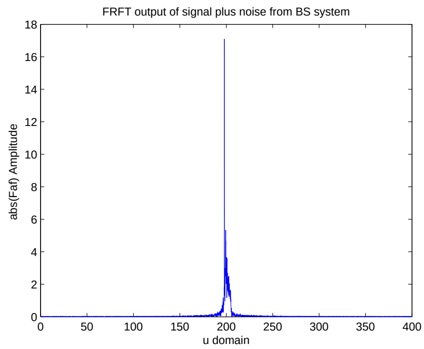

Figure 12: FRFT of BS output, input is signal plus 5dB white noise

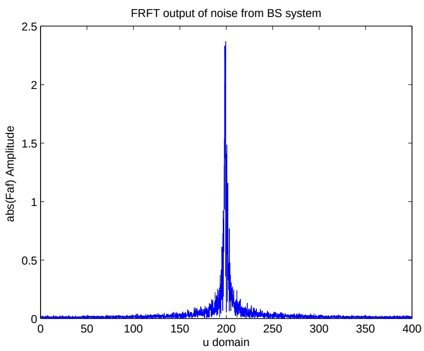

Figure 13: FRFT of BS output, input is 5dB white noise only

Compared with Figure 6, Figure 12 displays an apparent peak. Meanwhile, if comparing Figure 12 with Figure 5, we note that the maximum amplitude amplifies from nearly 10 to 17, that is just the role of stochastic resonance of BS system.

We then discuss the question by numerical result – what's the signal extent of signals which can be detected? We can use the variable G defined in equation (19) to measure the detection ability of BSFRFT and MBSFRFT. In order to get more accurate result of Runge-Kutta algorithm, we take step value $h = 0.01$ here. Under 100 times Monte Carlo simulation, the average $G - D$ curves of methods BSFRFT and MBSFRFT are shown in Figure 14 and Figure 15.

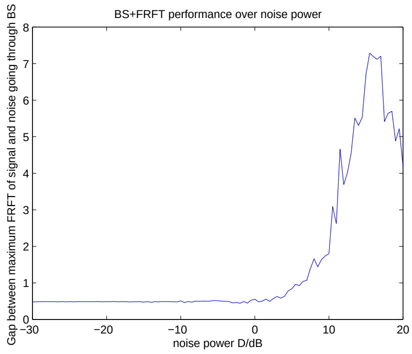

Figure 14: G of method BSFRFT versus noise power D

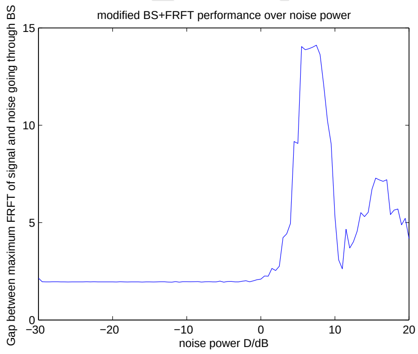

Figure 15: G of method MBSFRFT versus noise power D

From Figure 14, we note that G is nearly constant when noise is weak, and increases sharply when noise power is $10dB$, and then decreases gradually after noise power is larger than $16dB$, that is consistent with general SR non-monotonical tendency of BS system. Figure 14 and Figure 11 show when noise is weak (noise power is less than $5dB$), FRFT has better detection ability, but BSFRFT has better performance with the help of SR effect when noise power is larger than $10dB$. Moreover, from Figure 14 and Figure 15 we find that, there is a different interval from $3dB$ to $10dB$. It is a noise interval where noise output from BS system has DC component while LFM signal does not.

We take $D = 8dB$ as an example. Figure 16 and Figure 17 show the outputs from BS system, where input are LFM signal plus white noise and white noise only respectively. It is clear that noise output has DC component near equilibrium point 1 while signal does not. Figure 19 and Figure 20 show the FRFT outputs of noise directly and of noise with DC component removed. It is apparent that the modified method MBSFRFT is more valid in this situation.

5.2 Simulation of additive Gaussian colored noise

Similar to the case of white noise, we now discuss LFM signal detection when noise is Gaussian colored noise. Firstly, we use Runge-Kutta algorithm (11) to get the BS output $xc(t)$ where $\xi(n)$ is substituted by colored noise $c\xi(n)$. Secondly, we compute the FRFT output of $xc(t)$ and compare it with the detection threshhold.

We test two types of Gaussian colored noise. The first one is obtained by making Gaussian white noise go through a Butterworth low pass filter. Its time domain curves make little difference with Gaussian white noise, as shown in Figures 21 and 22 (SNR=$-20dB$). The LFM signal is also covered completely by noise like white noise. However, Figure 23 shows its power spectral density is different. We can also get the average $G - D$ curves of methods BSFRFT and MBSFRFT in the case of Gaussian colored noise in Figure 24 and Figure 25. It's nearly the same as the average $G - D$ curves in white noise case, only the maximum value here is smaller.

Another colored noise type is special, named pink noise or $1/f$ noise. Figure 26 shows its power spectral density (noise power is $10dB$). The average $G - D$ curves are shown in Figure 27 and Figure 28.

Correspondingly, we give the $G - D$ curve of FRFT in pink noise case in Figure 29. Due to the greater fluctuation of colored noise, FRFT and BSFRFT/MBSFRFT all have worse performance compared to white noise scene. When noise power is larger than $-10dB$, it is nearly impossible for FRFT to differentiate signal from noise. However, BSFRFT/MBSFRFT has a wider extent that it can tell signal from noise until noise power is larger than $0dB$.

6 Conclusions

This paper proposes a novel LFM weak signal detection algorithm based on BS system and FRFT. We also present an evaluation criteria to measure the ability of detection methods and apply it to comparing performance of FRFT and our novel method. Numerical simulations show that the method is effective for two cases – the LFM signal covered by white noise or colored noise (including pink noise).

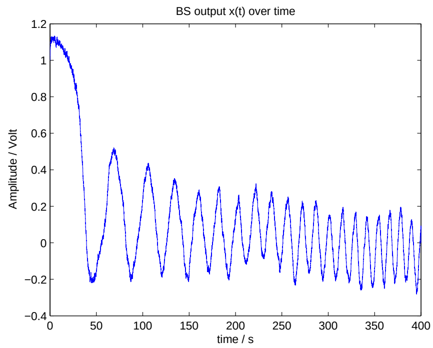

Figure 16: BS output of signal plus noise(D=8dB)

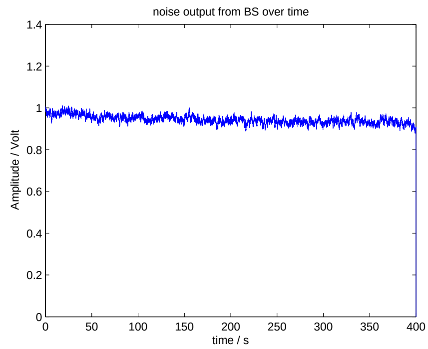

Figure 17: BS output of only noise(D=8dB)

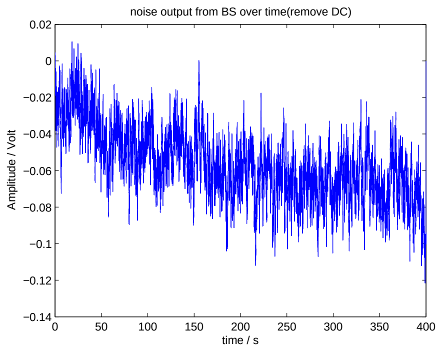

Figure 18: BS output of only noise(D=8dB,removed DC)

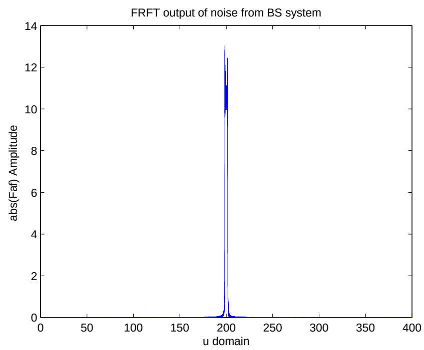

Figure 19: FRFT of noise from BS(D=8dB)

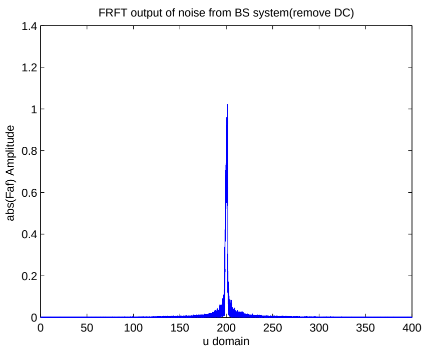

Figure 20: FRFT of noise from BS(D=8dB,removed DC)

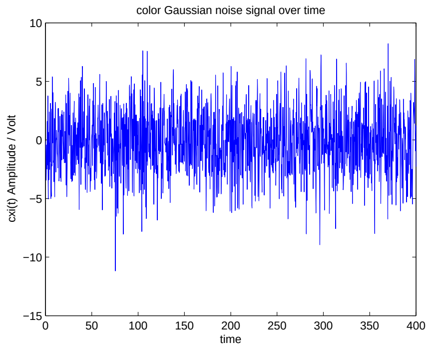

Figure 21: Gaussian colored noise

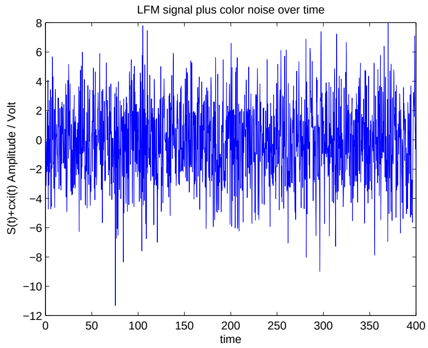

Figure 22: LFM signal plus Gaussian colored noise

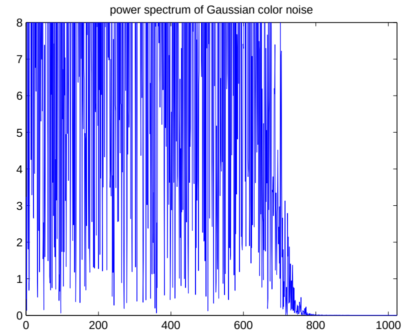

Figure 23: Power spectral density of Gaussian colored noise

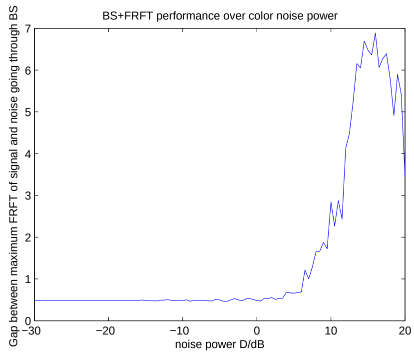

Figure 24: G of method BSFRFT versus colored noise power D

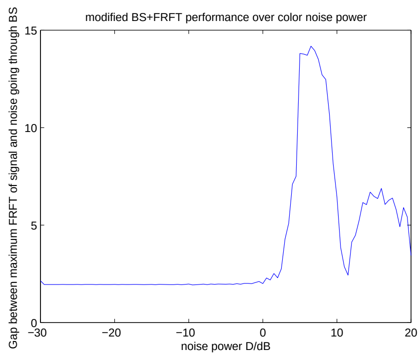

Figure 25: G of method MBSFRFT versus colored noise power D

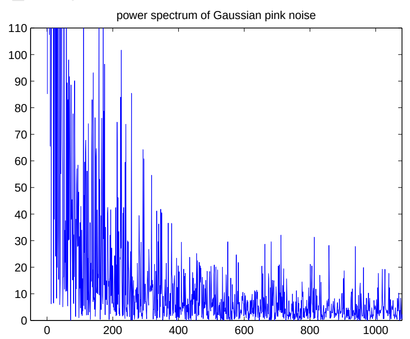

Figure 26: Power spectral density of pink noise

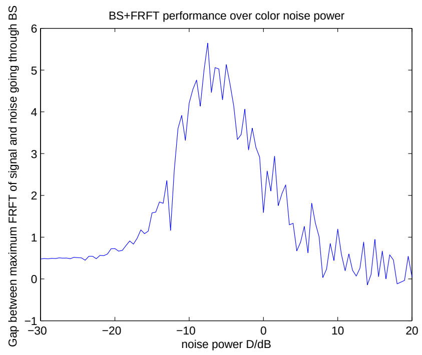

Figure 27: G of method BSFRFT versus pink noise power D

Figure 28: G of method MBSFRFT versus pink noise power D

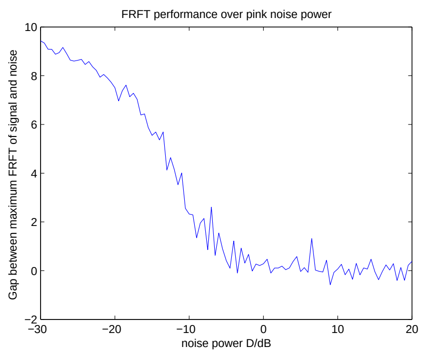

Figure 29: FRFT G versus pink noise power D

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